Quantum/Nuclear Physics

WORKSHEET #3

Name: ______________________

1. A bismuth isotope decays into thallium by emitting an alpha particle according to the following equation:

$$^{217}_{83}Bi \rightarrow Tl + {}^{4}_{2}He$$

a. Determine the atomic number Z and the mass number *A* of the thallium nuclei produced and enter your answers in the spaces provided below.

Z = ___81___ A = __213__

The mass of the alpha particle is 6.64×10^{-27} kg. Its measured kinetic energy is 6.09 MeV and its speed is much less than the speed of light.

b. Determine the momentum of the alpha particle.

$6.09\text{x}10^{6}$ eV · ($1.6\text{x}10^{-19}$ J/eV) = $9.744\text{x}10^{-13}$ J
$v = (2KE/m)^{\frac{1}{2}} = (2 \cdot 9.744\text{x}10^{-13}\text{ J} / 6.64\text{x}10^{-27}\text{ kg})^{\frac{1}{2}} = 1.713\text{x}10^{7}$ m/s
$p = mv = 6.64\text{x}10^{-27}\text{ kg} \cdot 1.713\text{x}10^{7}\text{ m/s} =$ $\boxed{1.14\text{x}10^{-19}\text{ kg·m/s}}$

c. Determine the kinetic energy of the recoiling thallium nucleus.

$p_{Tl} = p_{\alpha}$
(conservation of momentum - particles will have equal and opposite momentums)
$v_{Tl} = p_{\alpha}/m_{Tl} = 1.14\text{x}10^{-19}\text{ kg·m/s} / (213\text{ u} \cdot (1.66\text{x}10^{-27}\text{ kg/1 u})) = 3.22\text{x}10^{5}$ m/s
$KE = \frac{1}{2} mv^2 = 0.5 \cdot 213\text{ u} \cdot (1.66\text{x}10^{-27}\text{ kg/1 u}) \cdot (3.22\text{x}10^{5}\text{ m/s})^2 =$ $\boxed{1.84\text{x}10^{-14}\text{ J}}$

d. Determine the total energy released during the decay of 1 mole of bismuth 212:

$TE_{nucleas} = KE_{Tl} + KE_{\alpha} = 1.84\text{x}10^{-14}\text{ J} + 9.744\text{x}10^{-13}\text{ J} = 9.928\text{x}10^{-13}$ J
$TE_{mole} = TE_{nucleus} \cdot 6.02\text{x}10^{23}\text{ decays/mol} = 9.928\text{x}10^{-13}\text{ J} \cdot 6.02\text{x}10^{23} =$ $\boxed{5.98\text{x}10^{11}\text{ J}}$

2. The mass of He-4 is 4.002 60 u. The mass of a proton is 1.007 825 u and the mass of a neutron is 1.008 665 u. (a) Calculate the binding energy for He-4. (b) explain how this is involved in the release of energy during the process of fission.

a) mass defect = $2m_p + 2m_n - m_{He} = 2 \cdot 1.007825\text{ u} + 2 \cdot 1.008665\text{ u} - 4.00260$ u
= 0.03038 u or 0.03038 u · (931 MeV/c^2)/1 u = 28.28378 MeV/c^2
$E = mc^2 = (28.28378\text{ MeV}/c^2) \cdot c^2 =$ $\boxed{28.28378\text{ MeV}}$

b) When nuclei are split, the binding energy holding them together (the strong force) is released as energy. The binding energy can be calculated by using Einstein's $E=mc^2$ where "m" is the difference in mass between the individual particles making up the nucleus and the mass of the entire nucleus.

3. Let us assume that a typical house requires 2.0 x 10^3 kWh for a one month period. Let us also assume that there are 120 khouses in Wyoming. If one fission of a U-235 nucleus releases 208 MeV and 100% of this energy is converted into electricity, how many kg of uranium would be required to supply the energy needs of the people of our beloved state for a year?

2.0×10^3 kWh · 120 000 houses = 2.4×10^8 kWh
2.4×10^8 kWh · (1000 W/1 kW) · (3600 s/1 h) = 8.64×10^{14} W·s = 8.64×10^{14} J
8.64×10^{14} J/month · (12 months/1 y) = 1.0368×10^{16} J (energy required for the year)
208 MeV · (1 000 000 eV/1 MeV) · (1.6×10^{-19} J/1 eV) = 3.328×10^{-11} J (per fission)
U-235 has 235 g per mole, so there would be 1 mol of fissions per 0.235 kg:
3.328×10^{-11} J/fission · 6.02×10^{23} fissions/0.235 kg = 8.525345×10^{13} J/kg
1.0368×10^{16} J · 1 kg/8.525345×10^{13} J = 121.6 kg = $\boxed{122\text{ kg}}$

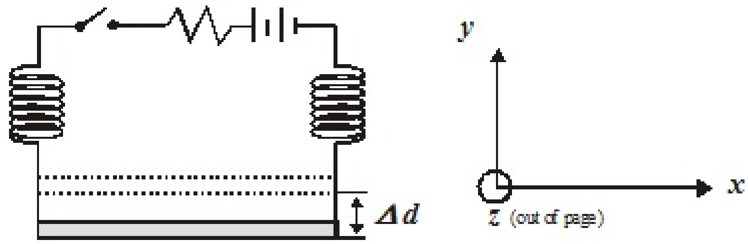
4. A rigid rod of mass ***m*** has a length of **l**. It is suspended from two identical springs of negligible mass as shown in the diagram below. The upper ends of the springs are fixed in place and the springs stretch a distance ***d*** under the weight of the suspended rod.

a. Determine the spring constant for each spring in terms of the other given quantities and fundamental constants.

a. $F_g = 2F_s$
$mg = 2kd$ so $\boxed{k = mg/2d}$

As shown below, the upper end of the springs are connected by a circuit branch containing a battery of emf $\mathscr{E}$ and a switch ***S***, so that a complete circuit is formed with the metal rod and springs. The circuit has a total resistance ***R***, represented by the resistor in the diagram below. The rod is in a uniform magnetic field, directed perpendicular to the page. The upper ends of the springs remain fixed in place and switch ***S*** is closed. When the system comes to equilibrium, the rod is lowered an additional distance of Δd.

b. What is the direction of the magnetic field relative to the coordinate axis shown on the right in the diagram?



c. Determine the magnitude of the field in terms of ***m***, l, ***d***, ***Δd***, $\mathscr{E}$, ***R***, and fundamental constants.

d. When the switch is suddenly opened, the rod oscillates. For these oscillations, determine the following quantities in terms of , ***d***, ***Δd***, and fundamental constants.
i. The period
ii. The maximum speed of the rod.

b. magnetic field is oriented in the +z direction (by right hand rule)
c. $I \ell B\sin\theta = 2k\Delta d)$
$I \ell B\sin 90° = 2mg\Delta d/2d = mg\Delta d/d$
$\mathscr{E} \ell B/R = mg\Delta d/d$
$\boxed{B = mg\Delta dR/d\mathscr{E}\ell}$
d) i. $T_S = 2\pi(m/k)^{\frac{1}{2}} = 2\pi(m/(mg/d))^{\frac{1}{2}} = 2\pi(d/g)^{\frac{1}{2}}$
ii. $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$
$v = (kx^2/m)^{\frac{1}{2}} = ((mg/d)\Delta d^2/m)^{\frac{1}{2}} = \boxed{\Delta d(g/d)^{\frac{1}{2}}}$

5. Complete the following nuclear reactions and indicate the type of decay reaction:

$^{7}_{3}Li + ^{1}_{1}H \rightarrow ^{4}_{2}He + ^{4}_{2}He$ (proton bombardment with alpha c

$^{56}_{26}Fe \rightarrow ^{56}_{27}Co + ^{0}_{-1}e$ (beta decay)

6. What wavelength of light would have to be incident on sodium metal if it is to emit electrons with a maximum speed of 1.00×10^6 m/s? (Work function for sodium is 2.46 eV)

$hc/\lambda = \Phi + \frac{1}{2} mv^2$

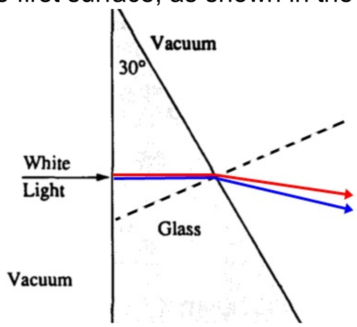
$\lambda = hc/(\Phi + \frac{1}{2} mv^2)$

$= 1.24\text{x}10^3 \text{ eV}\cdot\text{nm}/(2.46 \text{ eV} + 0.5 \cdot 9.11\text{x}10^{-31} \text{ kg} \cdot (1.00\text{x}10^6 \text{ m/s})^2 \cdot (1 \text{ eV}/1.6\text{x}10^{-19} \text{ J}))$

$= 1.24\text{x}10^3 \text{ eV}\cdot\text{nm}/(2.46 \text{ eV} + 2.847 \text{ eV}) =$ **234 nm**

7. The glass prism shown below has an index of refraction that depends on the wavelength of the light that enters it. The index of refraction is 1.50 for red light of wavelength 700 nanometers (700×10^{-9} meter) in vacuum and 1.60 for blue light of wavelength 480 nanometers in vacuum. A beam of white light is incident from the left, perpendicular to the first surface, as shown in the figure, and is dispersed by the prism into its spectral components.

	Wavelength in Vacuum	Index of Refraction of Glass
Red Light	700 nm	1.5
Blue Light	480 nm	1.6



a. Determine the speed of the blue light in the glass.

a. $n_{blue} = c/c_{blue}$ so $c_{blue} = c/n = 3.00\text{x}10^8 \text{ m/s} / 1.6 = 1.875\text{x}10^8 \text{ m/s} =$ **$1.88\text{x}10^8$ m/s**

b. Determine the wavelength of the red light in the glass.

b. $n_1\lambda_1 = n_2\lambda_2$ $\lambda_2 = n_1\lambda_1/n_2 = 1 \cdot 700 \text{ nm}/1.5 =$ **467 nm**

c. Determine the frequency of the red light in the glass.

c. $f = c/\lambda = 3.00\text{x}10^8 \text{ m/s} / 700\text{x}10^{-9} \text{ m} =$ **$4.286\text{X}10^{14}$ Hz or 428.6 THz**

d. i. What is the angle of refraction for the blue and red light incident on the front surface?
ii. Calculate the angle of refraction for the blue and red light incident on the far surface.
iii. On the figure above, sketch the approximate paths of both these rays as they pass through the glass and back out into the vacuum. Ignore any reflected light. Clearly show the change in direction of the rays, if any, at each surface and be sure to distinguish carefully any differences between the paths of the red and the blue beams.

d. $n_1\sin\theta_1 = n_2\sin\theta_2$ so $\theta_2 = \sin^{-1}(n_1\sin\theta_1/n_2)$

i. Both are entering at an angle of 0° (angle of incidence) -- right along the normal line, so there is no change to the angle of refraction. Refraction angle for both will be 0°.

ii. Blue: $\theta_2 = \sin^{-1}(1.6 \cdot 0.5/1) =$ **53.1°**

Red: $\theta_2 = \sin^{-1}(1.5 \cdot 0.5/1) =$ **48.6°**

iii. See figure above.